The role of five-quark components in gamma decay of the $\Delta(1232)$

Q. B. Li* and D. O. Riska†

Helsinki Institute of Physics, POB 64,

00014 University of Helsinki, Finland

(Dated: February 9, 2008)

Abstract

An admixture of 10 - 20% of $qqqq\bar{q}$ components in the $\Delta(1232)$ resonance is shown to reduce the well known underprediction by the qqq quark model of the decay width for $\Delta(1232) \to N\gamma$ decay by about half and that of the corresponding helicity amplitudes from a factor ~ 1.7 to ~ 1.5. The main effect is due to the quark-antiquark annihilation transitions: $qqqq\bar{q} \to qqq\gamma$, the consideration of which brings the ratio $A_{\frac{3}{2}}/A_{\frac{1}{2}}$ and consequently the $E2/M1$ ratio R_{EM} into agreement with the empirical value. Transitions between the $qqqq\bar{q}$ components in the resonance and the nucleon: $qqqq\bar{q} \to qqqq\bar{q}\gamma$, are shown to enhance the calculated decay width by only a few percent, as long as the probability of the $qqqq\bar{q}$ component of the proton and the $\Delta(1232)$ is at most 20%. The transitions $qqqq\bar{q} \to qqqq\bar{q}\gamma$ between the $qqqq\bar{q}$ components in the $\Delta(1232)$ and the proton do not lead to a nonzero value for R_{EM}.

*ligb@pcu.helsinki.fi

†riska@pcu.helsinki.fi

I. INTRODUCTION

The 3 valence quark model for the baryons does not provide a quantitatively satisfactory description of the electromagnetic and strong decay widths of the lowest energy nucleon resonances. Even in refined versions of that model the calculated decay width of the $\Delta(1232)$ is typically less than half of the empirical value, while those of the $N(1440)$ and the $\Delta(1600)$ are even smaller [1, 2]. Explicit coupled channel treatments of the interacting $\pi - N - \Delta$ system [3, 4] show that this problem is likely to arise from coupling to explicit pion degrees of freedom, which are missing in the qqq valence quark model. This suggests that the quark model be extended to include explicit sea-quark ($q\bar{q}$) configurations. The presence of such sea-quark configurations in the proton has in fact been experimentally demonstrated [5, 6, 7, 8].

Here such an extension of the valence quark model is made to include those $qqqq\bar{q}$ components in the proton and the $\Delta(1232)$ wave function, which are likely to require the lowest excitation energy, in order to study the effect of these $qqqq\bar{q}$ components on the calculated electromagnetic decay rates of the $\Delta(1232)$. As in a previous study of the effect of $qqqq\bar{q}$ components on the calculated pion decay rate of the $\Delta(1232)$, it is found that transitions between the $qqqq\bar{q}$ components in the resonances and in the nucleon themselves leads to modifications of only a few percent [9]. On the other hand the direct quark-antiquark annihilation transitions between the $qqqq\bar{q}$ components in the resonances and the qqq component of the proton is significant.

The confining and hyperfine interactions between the quarks can also trigger such quark-antiquark annihilation transitions. In the case of pion decay the confining interaction leads to an enhancement of the net effect of annihilation transitions on the calculated decay rates. Here it is found to have the opposite effect in the case of the electromagnetic decays, unless the interaction potential is attractive at short range . The magnitude of the confinement triggered annihilation transitions is estimated with two schematic models (linear and harmonic) for the confining interaction. Given the opposite effects of the confinement triggered annihilation transitions in pion and electromagnetic decay, the conclusion is that this effect should be of minor net significance.

It is found that the effect of the direct annihilation transitions on the calculated helicity amplitudes for gamma decay of $\Delta(1232)$ is to bring them closer to the empirical values. A

probability of $\sim 20\%$ for the $qqqq\bar{q}$ component in the $\Delta(1232)$ leads to a reduction of the underprediction of ~ 1.7 in the valence quark model to ~ 1.5. This leads to a corresponding reduction of the underprediction of the radiative width by about half.

The quark-antiquark annihilation transitions bring the calculated helicity amplitude ratio $A_{3/2}/A_{1/2}$ into agreement with the empirical ratio. As a consequence they lead to a non-vanishing value for the calculated $E2/M1$ ratio R_{EM}, which falls within the empirical range.

The paper is structured in the following way. In section II the effect of transitions between the $qqqq\bar{q}$ components wave functions of proton and $\Delta(1232)$ in $qqqq\bar{q}$ configurations are calculated. In Section III the corresponding annihilation transitions $qqqq\bar{q} \to qqq\gamma$ are considered. Section IV contains a concluding discussion.

II. TRANSITIONS BETWEEN $qqqq\bar{q}$ COMPONENTS IN $\Delta^+ \to p\gamma$ DECAY

Consider $\Delta^+ \to p\gamma$ decay that arises from the direct electromagnetic coupling to constituent quarks: $qq\gamma$. To lowest order in the photon momentum the transition amplitude obtained from the electromagnetic γ_μ coupling for pointlike quarks is then

$$T_i = \frac{e_i}{2m}\sigma_{i-}\sqrt{k_\gamma}, \tag{1}$$

where e_i and m are the electric charge and mass of the quark that emits the photon, respectively. The momentum of the final right-handed photon is taken to be in the direction of the z-axis: $\vec{k} = (0, 0, k_\gamma)$ with k_γ=259 MeV in the center of mass frame of $\Delta(1232)$.

The $\Delta^+ \to p\gamma$ transition is described by the two independent helicity amplitudes:

$$\begin{aligned} A_{\frac{3}{2}} &= \langle p, \frac{1}{2}\,\frac{1}{2} \mid \sum_i^{n_q} T_i \mid \Delta^+, \ \frac{3}{2}\,\frac{3}{2}\rangle \,, \\ A_{\frac{1}{2}} &= \langle p, \ \frac{1}{2}\ -\frac{1}{2} \mid \sum_i^{n_q} T_i \mid \Delta^+, \ \frac{3}{2}\,\frac{1}{2}\rangle \,. \end{aligned} \tag{2}$$

These represent the helicity components 3/2 and 1/2 of the $\Delta(1232)$ on the direction of the photon momentum. In eq. (2) n_q is the number of constituent quarks.

The spatial wave function of the quarks in the spatially symmetric ground state will be schematically described by the harmonic oscillator wave function:

$$\varphi_0(\xi_i) = (\frac{\omega_3^2}{\pi})^{3/4}\, e^{-\xi_i^2\,\omega_3^2/2} \,. \tag{3}$$

The scale of the oscillator parameter ω_3 may be set by the empirical radius of the proton as $\omega_3 = 1/r_p \simeq 225$ MeV. In (3) $\vec{\xi_i}$, i=1, 2, are the standard Jacobi coordinates of the 3-quark system. The helicity amplitudes for $\Delta^+ \to p\gamma$ decay in the conventional qqq configuration are then:

$$
\begin{aligned}
A^{(3q)}_{\frac{3}{2}} &= -\frac{\sqrt{6}}{3}\frac{e}{2m}\sqrt{k_\gamma}\,(1-\frac{{k_\gamma}^2}{6\omega_3^2})\,,\\
A^{(3q)}_{\frac{1}{2}} &= -\frac{\sqrt{2}}{3}\frac{e}{2m}\sqrt{k_\gamma}\,(1-\frac{{k_\gamma}^2}{6\omega_3^2}).
\end{aligned}
\tag{4}
$$

Here m is again the constituent quark mass. These expressions yield the usual quark model value for the ratio of the helicity amplitudes: $A^{(3q)}_{\frac{3}{2}}/A^{(3q)}_{\frac{1}{2}} = \sqrt{3}$. If the constituent quark mass is taken to be 340 MeV, these expressions lead to the values $A^{(3q)}_{\frac{1}{2}} = -0.083/\sqrt{\text{GeV}}$ and $A^{(3q)}_{\frac{3}{2}} = -0.143/\sqrt{\text{GeV}}$. These values are smaller by factors 1.6 and 1.8, respectively, than the corresponding experimental values $A_{\frac{1}{2}} = -0.135 \pm 0.006/\sqrt{\text{GeV}}$ and $A_{\frac{3}{2}} = -0.255 \pm 0.008/\sqrt{\text{GeV}}$ [10].

Consider now $qqqq\bar{q}$ admixtures in the proton and the Δ^+. Positive parity demands that these have to be $P-$wave states. The spin dependence of the hyperfine interaction between the quarks implies that the $qqqq\bar{q}$ configurations that have the lowest energy, and which are most likely to form notable admixtures in the baryon states, are those that have the most antisymmetric $qqqq$ configurations, which are compatible with the requirement of overall antisymmetry. In the case of the nucleon this state has the mixed spin-flavor symmetry $[4]_{FS}[22]_F[22]_S$ and in the case of the $\Delta(1232)$ the mixed spin-flavor symmetry $[4]_{FS}[31]_F[31]_S$ [11].

The $qqqq\bar{q}$ component in the proton, with a $qqqq$ configuration with spin-flavor symmetry $[4]_{FS}[22]_F[22]_S$ has mixed spatial symmetry $[31]_X$, and may be represented by the wave function:

$$
\begin{aligned}
\psi_p(M_S) = &\frac{A_{p5}}{\sqrt{2}}\sum_{a,b}\sum_{m,s}(1,1/2,m,s|\,1/2,M_S)\,C^{[1111]}_{[211]a,[31]a}\\
&[211]_C(a)\,[31]_{X,m}(a)\,[22]_F(b)\,[22]_S(b)\,\bar{\chi}_s\,\phi(\{r_i\})\,.
\end{aligned}
\tag{5}
$$

Here M_S denotes the spin-z component of the state and A_{p5} is the amplitude of the configuration. The symbol $C^{[1111]}_{[211]a,[31]a}$ is a S_4 Clebsch-Gordan coefficient. The color, space and flavor-spin wave functions of the $qqqq$ subsystem have here been denoted by their Young patterns respectively. The sum over a runs over the 3 configurations of the $[211]_C$ and $[31]_X$

representations of S_4, and the sum over b runs over the 2 configurations of the [22] representation of S_4 respectively [12]. Note that as the isospin of the $qqqq$ of the $[22]_F$ configuration is 0, the antiquark can only be a $\bar{d}$ quark in this configuration.

The corresponding $qqqq\bar{q}$ configuration in the $\Delta(1232)$, with the flavor-spin symmetry $[4]_{FS}[31]_F[31]_S$ in the $qqqq$ configuration may be represented by the wave function:

$$\psi^{(J)}_{\Delta^+}(M_S) = \frac{A^{(J)}_{\Delta 5}}{\sqrt{3}} \sum_{a,b} \sum_{m,s,M,j;T,t} (1,1,m,M|\,J,j)(J,1/2,j,s|\,3/2,M_S)\, C^{[1111]}_{[211]a,[31]a}$$
$$(1,1/2,T,t|3/2,1/2)[211]_C(a)\,[31]_{X,m}(a)\,[31]_{F,T}(b)\,[31]_{S,M}(b)\,\bar{\chi}_{t,s}\,\phi(\{r_i\})\,. \tag{6}$$

Here J denotes the total angular momentum of the $qqqq$ system, which takes the values 1 and 2, and $A^{(J)}_{\Delta 5}$ is the amplitude of the configuration in the $\Delta(1232)$. The sum over a again runs over the 3 configurations of the $[211]_C$ and $[31]_X$ representations of S_4. Here the sum over b runs over the 3 configurations of the [31] representation. Here the isospin-z component of the 4-quark state is denoted T and that of the antiquark t, which results in the quark combination $uudd\bar{d}$ for T=0, t=1/2 and $uuud\bar{u}$ for T=1, t=$-1/2$. Since there is no isospin flip in the transition operator (1), only the five quark configuration $uudd\bar{d}$ in Δ^+ contributes to the direct transition $\Delta^+ \to p\gamma$.

The orbital wave function of the P-shell $qqqq$ states $[31]_X$ in eqs. (5) and (6) are described by the product of the S-wave and P-wave harmonic oscillator functions:

$$\tilde{\varphi}_0(\xi_i) = (\frac{\omega_5^2}{\pi})^{3/4}\, e^{-\xi_i^2\,\omega_5^2/2}\,, \qquad \tilde{\varphi}_{1m}(\xi_i) = \sqrt{2}\omega_5 \xi_{i,m}\varphi_0(\xi_i)\,. \tag{7}$$

Here the oscillator parameter ω_5 is that for the $qqqq\bar{q}$ system. The operators ξ_i, i=1..3, are the standard Jacobi coordinates for the five-quark system in the spherical basis [11].

The calculation of helicity amplitudes for the transition between the $qqqq\bar{q}$ components in proton and Δ is straightforward and leads to:

$$\begin{aligned} A^{(5q)}_{\frac{3}{2}} &= -\frac{2\sqrt{3}}{9}(\delta_{J1} + \sqrt{5}\delta_{J2})\frac{e}{2m}\sqrt{k_\gamma}\;(1 - \frac{{k_\gamma}^2}{5\omega_5^2})\,, \\ A^{(5q)}_{\frac{1}{2}} &= -\frac{2}{9}(\delta_{J1} + \sqrt{5}\delta_{J2})\frac{e}{2m}\sqrt{k_\gamma}\;(1 - \frac{{k_\gamma}^2}{5\omega_5^2})\,. \end{aligned} \tag{8}$$

From eqs. (4) and (8) one obtains the ratio between the helicity amplitudes for direct transition when the $qqqq\bar{q}$ configurations in proton and $\Delta^+(1232)$ are included to be:

$$\frac{A_{\frac{3}{2}}}{A_{\frac{1}{2}}} = \frac{A_{p3}A_{\Delta 3}A^{3q}_{\frac{3}{2}} + A_{p5}A^{(J)}_{\Delta 5}A^{5q}_{\frac{3}{2}}}{A_{p3}A_{\Delta 3}A^{3q}_{\frac{1}{2}} + A_{p5}A^{(J)}_{\Delta 5}A^{5q}_{\frac{1}{2}}} = \sqrt{3}, \tag{9}$$

which is the standard quark model result. Here A_{p3}, A_{p5} are the amplitudes for the qqq and $qqqq\bar{q}$ components of the proton and the $\Delta(1232)$ respectively, and $A_{\Delta 3}$, $A_{\Delta 5}^{(J)}$ are the amplitudes for the corresponding components of the $\Delta(1232)$.

The magnetic dipole M_1 and electric quadrupole E_2 moment contributions to $\Delta^+ \to p\gamma$ decay are related to the helicity amplitudes as [10]

$$M_1 = -\frac{1}{2\sqrt{3}}\,(3A_{\frac{3}{2}} + \sqrt{3}A_{\frac{1}{2}})\,,$$
$$E_2 = \frac{1}{2\sqrt{3}}\,(A_{\frac{3}{2}} - \sqrt{3}A_{\frac{1}{2}}). \qquad (10)$$

Since the contribution from the direct transitions between $qqqq\bar{q}$ components of the $\Delta(1232)$ and the nucleon leaves the calculated ratio $A_{\frac{3}{2}}/A_{\frac{1}{2}}$ unchanged from the value $\sqrt{3}$ given by the qqq configuration with spatially symmetric wave functions(9), they leave the $E2$ amplitude unchanged at 0.

While direct transitions between the $qqqq\bar{q}$ components in proton and Δ do not change the calculated value for the E_2/M_1 ratio for $\Delta^+ \to p\gamma$ decay, they do affect the calculated decay width. In terms of the helicity amplitudes, the decay width is given by [10],

$$\Gamma = \frac{k_\gamma^2}{\pi}\,\frac{2M_p}{(2J+1)M_\Delta}[|A_{\frac{3}{2}}|^2 + |A_{\frac{1}{2}}|^2], \qquad (11)$$

where M_p and M_Δ are the masses of proton and the Δ, respectively, and $J = 3/2$ is the spin of Δ. By taking into account the normalization of the wave functions of proton and $\Delta(1232)$ in both the qqq and $qqqq\bar{q}$ configurations, the enhancement of the calculated decay width that arises from the direct transitions between the $qqqq\bar{q}$ components is:

$$\delta = \frac{\Gamma}{\Gamma_{3q}} = \frac{\sum_{\lambda=1/2,3/2} |A_{p3}A_{\Delta 3}A_\lambda^{(3q)} + A_{p5}A_{\Delta 5}^{(J)}A_\lambda^{(5q)}|^2}{|A_{\frac{3}{2}}^{(3q)}|^2 + |A_{\frac{1}{2}}^{(5q)}|^2}. \qquad (12)$$

With the present wave function model, (4), (8), this leads to the expression:

$$\delta = |A_{p3}A_{\Delta 3} + \frac{\sqrt{2}}{3}A_{p5}(A_{\Delta 5}^{(1)} + \sqrt{5}A_{\Delta 5}^{(2)})|^2\,, \qquad (13)$$

if the radii of the 3- and the 5-quark components are taken to be equal, so that

$$\omega_5 = \sqrt{\frac{6}{5}}\omega_3\,. \qquad (14)$$

It is worth noting that the condition (14) is not necessary for the eq. (9). The numerical effect of the 5 quark components is small even when the probability of the $qqqq\bar{q}$ components

of the proton and the $\Delta(1232)$ are larger than 10%. With 10% probability for the $qqqq\bar{q}$ component in the nucleon and $\Delta(1232)$, in which the proportion of the $J=1$ and $J=2$ $qqqq$ states in $\Delta(1232)$ are assumed to be 50% and 50%, respectively, the effect of the $qqqq\bar{q}$ component is to enhance the calculated values of both helicity amplitudes by a factor $\sqrt{\delta} \sim 1.01$, and the decay width by a factor $\delta \sim 1.02$. With the enhancement factor 1.01 the helicity amplitude $A_{\frac{1}{2}}$ is smaller than the empirical value by a factor 1.6, and the calculated value of $A_{\frac{3}{2}}$ smaller by a factor 1.7 than the corresponding empirical value. Hence the net effect of the transition between the $qqqq\bar{q}$ component in the $\Delta(1232)$ and proton is to increase the decay width by only a few percent at most.

III. $q\bar{q}$ ANNIHILATION TRANSITIONS IN $\Delta^+ \to p\gamma$ DECAY

A. Direct annihilation transitions

The Dirac (γ_μ) coupling for pointlike quarks used in previous section leads to following the $q\bar{q} \to \gamma$ transition operator for the transition $\Delta^+ \to p\gamma$ illustrated in in Fig. 1:

$$T_a = \sum_{i=1}^{4} e_i \sigma_{i-} \frac{1}{\sqrt{k_\gamma}}, \tag{15}$$

where e_i is the electric charge of the quark that annihilates the anti-quark and σ_{i-} is the spin lowering operator. The σ_{i-} in eq. (15) requires the annihilating quark to have the spin-z component 1/2, which, in combination with the anti-quark with spin-z component 1/2, produces the final photon with angular moment $L=1, L_z=1$. Note there is no contribution from transitions of the reverse type $qqq \to qqqq\bar{q}$.

The helicity amplitudes for the $qqqq\bar{q} \to qqq+\gamma$ transition are obtained as matrix elements of the operator (15) between the proton in qqq configuration and Δ in $qqqq\bar{q}$ configuration (6). Note that both of the quark combinations $uudd\bar{d}$ and $uuud\bar{u}$ in the Δ^+ contribute to the $qqqq\bar{q} \to qqq+\gamma$ transition through $d\bar{d} \to \gamma$ and $u\bar{u} \to \gamma$, respectively. This leads to the following factor in spin-flavor-color (SFC) space:

$$C_{SFC}^{3/2} = -\frac{4\sqrt{5}}{45} A_{p3} (\sqrt{5} A_{\Delta 5}^{(1)} + A_{\Delta 5}^{(2)}) \,,$$

$$C_{SFC}^{1/2} = -\frac{8\sqrt{15}}{135} A_{p3} A_{\Delta 5}^{(2)} \,. \tag{16}$$

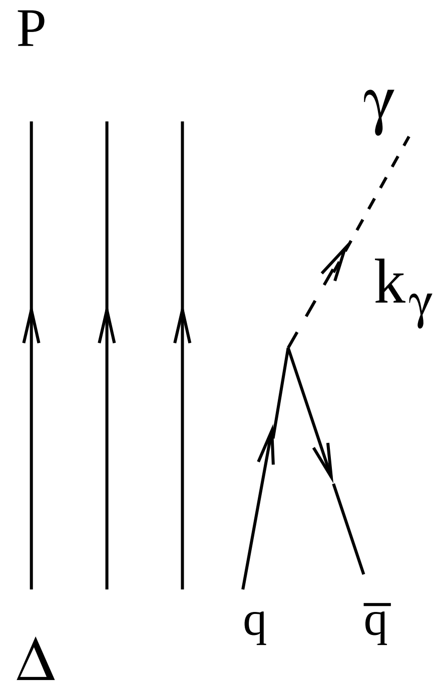

FIG. 1: Direct $qqqq\bar{q} \to qqq\gamma$ annihilation process

Here a factor 4 standing for the number of annihilating quark pairs has been multiplied and the normalization factors of the proton wave function in qqq configuration and Δ wave function in $qqqq\bar{q}$ configuration are explicit. It may be seen from eq. (16) that the $P-$shell $qqqq$ configuration in Δ does not contribute to the decay amplitudes with helicity 1/2. The matrix element in orbital space may be approximately evaluated with a power series expansion in k_γ, with the result [9]:

$$\langle T\rangle \simeq (\frac{\omega_3\omega_5}{\omega^2})^3 \frac{k_\gamma}{\omega_5}\frac{\sqrt{2}}{4}(1-\frac{3}{20}\frac{k_\gamma^2}{\omega_5^2})\,. \tag{17}$$

Here the normalization factor $(\omega_3\omega_5/\omega^2)^3$, with $\omega=\sqrt{(\omega_3^2+\omega_5^2)/2}$, comes from the different values of the size parameters ω_3 in eq. (3) and ω_5 in eq. (7).

The helicity amplitudes for the direct annihilation process are obtained by taking the product of the matrix elements in SFC space (16) and orbital space (17) as

$$\begin{aligned} A_{a\frac{3}{2}} &= -\frac{\sqrt{10}}{45}A_{p3}(\sqrt{5}A_{\Delta5}^{(1)}+A_{\Delta5}^{(2)})(\frac{\omega_3\omega_5}{\omega^2})^3\frac{e\sqrt{k_\gamma}}{\omega_5}(1-\frac{3}{20}\frac{k_\gamma^2}{\omega_5^2}) \\ A_{a\frac{1}{2}} &= -\frac{2\sqrt{30}}{135}A_{p3}A_{\Delta5}^{(2)}(\frac{\omega_3\omega_5}{\omega^2})^3\frac{e\sqrt{k_\gamma}}{\omega_5}(1-\frac{3}{20}\frac{k_\gamma^2}{\omega_5^2})\,. \end{aligned} \tag{18}$$

As the ratio of these amplitudes differs from $\sqrt{3}$ they lead to a non-vanishing value for the $E2/M1$ ratio.

Assuming again that the probability of the $qqqq\bar{q}$ configuration in both the proton and the $\Delta(1232)$ is 10%, in which the $J = 1$ and $J = 2$ $qqqq$ states of the $\Delta(1232)$ are assumed to have the proportion of 50% and 50%, respectively, the direct annihilation transition leads to an enhancement of the calculated value for the helicity amplitudes $A_{\frac{1}{2}}$ and $A_{\frac{3}{2}}$ factors 1.01 and 1.08 respectively. When these enhancement factors are combined with those that arise from transitions between the $qqqq\bar{q}$ components, the combined enhancement factor for the calculated value of the helicity amplitude $A_{\frac{1}{2}}$ is 1.12 and that for the amplitude $A_{\frac{3}{2}}$ is 1.18. When the standard qqq quark model values, -0.083/$\sqrt{\text{GeV}}$ and -0.143/$\sqrt{\text{GeV}}$, for these two amplitudes are multiplied by the corresponding enhancement factors, the net calculated values for the two amplitudes become $A_{\frac{1}{2}} = -0.093/\sqrt{\text{GeV}}$ and $A_{\frac{3}{2}} = -0.171/\sqrt{\text{GeV}}$, respectively. The first of these two values is smaller by a factor 1.4 than the corresponding empirical value, and the second is smaller by a factor 1.5 than the empirical value. The calculated ratio of the two helicity amplitudes, $A_{\frac{3}{2}}/A_{\frac{1}{2}} = 1.84$ is however considerably closer to the empirical value 1.89 than the value $\sqrt{3} \simeq 1.73$ that is obtained in the qqq and $qqqq\bar{q}$ quark model. Assuming a 10% probability of the $qqqq\bar{q}$ component in the proton and 20% in the $\Delta(1232)$, with equal proportion of the $J = 1$ and $J = 2$ $qqqq$ states in $\Delta(1232)$, the calculated helicity amplitudes increase to $A_{\frac{1}{2}} = -0.096/\sqrt{\text{GeV}}$ and $A_{\frac{3}{2}} = -0.180/\sqrt{\text{GeV}}$. These values lead to the ratio $A_{\frac{3}{2}}/A_{\frac{1}{2}} = 1.88$, which very close to the empirical value, and to an enhancement of the calculated decay width by 1.5.

B. Annihilation with quark-quark confinement interactions

Quark-antiquark annihilation transitions can be triggered by the interactions between the quarks in the baryons. The most obvious such triggering interaction is the confining interaction. Recently it has been noted that the confining interaction may contribute significantly to the calculated pion decay width of the Δ [9]. A similar effect naturally also should be expected in the case of the transition $\Delta^+ \to p\gamma$ (Fig. 2). To lowest order in the quark momenta the amplitude for this confinement triggered annihilation mechanism may, in the case of a linear confining interaction, be derived by making the following replacement

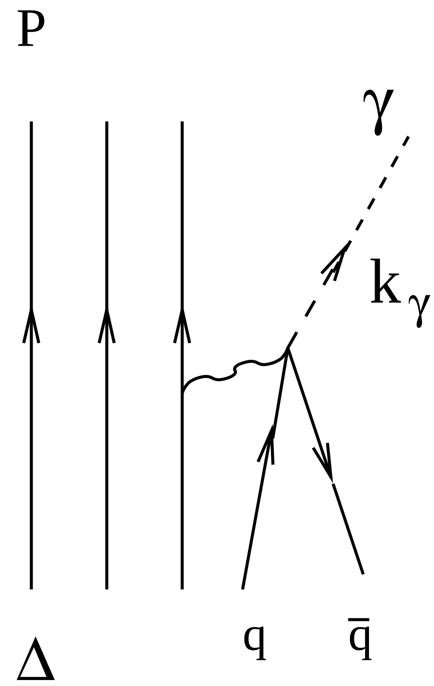

FIG. 2: Confinement induced $qqqq\bar{q} \to qqq\gamma$ annihilation process

in the transition operator for direct annihilation (15):

$$e_i\sigma_{i-} \to e_i\sigma_{i-}(1 - \frac{cr_{ij} - b}{2m})\,. \tag{19}$$

Here c is the string tension, r_{ij} is the distance between the two quarks that interact by the confining interaction and b is a constant, which shifts the zero point of the confinement. This replacement applies both in the case of scalar and vector coupled confinement. If the confining interaction is assumed to have the color coupling $\vec{\lambda}_i^C \cdot \vec{\lambda}_j^C$, the string tension c should be the same for all the qq and $q\bar{q}$ pairs in the $qqqq\bar{q}$ system, and half of the value for quark pairs in three-quark systems [16]. Here we use the value $c = 280$ MeV/fm [9].

From this expression it follows that for $b > 0$, which implies that the effective confining potential is negative at short range, and which is suggested by phenomenological study of the D and D_s meson spectra [13], the confining interaction reduces the net annihilation amplitude. This situation is similar to that in electromagnetic decay of heavy quarkonia and heavy light mesons [14, 15].

The orbital matrix element of the annihilation process with the linear interaction $-(cr -$

$b)/2m$ between quarks is given by [9]

$$\langle T_{conf}\rangle = -(\frac{\omega_3\omega_5}{\omega^2})^3\frac{c}{m\omega}\,(\frac{k_\gamma}{\omega_5})\,\frac{128\sqrt{15}}{375\pi}\,K(k_\gamma)\,. \tag{20}$$

Here the function $K(q)$ is defined as

$$K(q) = \omega_5^5\int_0^\infty d\xi\,\xi^4\,\frac{j_1(\beta q\xi)}{\beta q\xi}\,e^{-\alpha^2\xi^2}\,k(\omega\xi)\,. \tag{21}$$

The constants α and β in this expression are defined as $\alpha = 2\omega_5/\sqrt{5}$ and $\beta = 2\sqrt{3}/5$, respectively. The function $k(y)$ is defined as:

$$k(y) = \int_0^\infty dx x^2 e^{-x^2}\int_{-1}^1 dz\{\sqrt{x^2-2\sqrt{2}xzy+2y^2} - \frac{\sqrt{6}}{2}\frac{b\omega}{c}\}\,. \tag{22}$$

A numerical calculation gives $K(k_\gamma)$=0.71 at k_γ=259 MeV in the case $b = 0$.

The SFC matrix element of the annihilation process with confinement is the same as that of the direct annihilation given by eq. (16) and, combining with the orbital matrix element in eq. (20), one obtains the helicity amplitudes expressions:

$$\begin{aligned} A^c_{a\frac{3}{2}} &= \frac{512\sqrt{3}}{1125\pi}A_{p3}(\sqrt{5}A^{(1)}_{\Delta 5} + A^{(2)}_{\Delta 5})\;(\frac{\omega_3\omega_5}{\omega^2})^3\frac{c}{m\omega}\,\frac{e\sqrt{k_\gamma}}{\omega_5}K(k_\gamma) \\ A^c_{a\frac{1}{2}} &= \frac{1024}{1125\pi}A_{p3}A^{(2)}_{\Delta 5}\;(\frac{\omega_3\omega_5}{\omega^2})^3\frac{c}{m\omega}\,\frac{e\sqrt{k_\gamma}}{\omega_5}K(k_\gamma)\,, \end{aligned} \tag{23}$$

where an overall factor 3 has been inserted for the 3 similar interacting processes of annihilating the antiquark.

Due to the opposite sign of the amplitudes in eq. (23) to that in eq. (4), (8), (18), the confinement triggered annihilation reaction reduces the calculated values of the helicity amplitudes, and increases the disagreement with the empirical values. The quantitative importance of this effect does of course depend on the value of the constant b. If this is taken to be $\sim$ 480 MeV, the matrix element of the amplitude of the confinement triggered annihilation transition vanishes. The contribution of the annihilation transition triggered by linear confinement to the $\Delta(1232) \to p\gamma$ decay is therefore insignificant when the constant in the linear confining interaction is taken to be in the range by $350 - 500$ MeV (Table I). This range is only slightly larger than the values employed in the literature on heavy flavor spectroscopy [15].

To have an estimate of the theoretical uncertainty of the magnitude of the contribution of the confinement triggered annihilation process, we also consider the case of harmonic

TABLE I: Calculated helicity amplitudes, the ratio between the helicity amplitudes and the enhancement of calculated electromagnetic decay width from the qqq quark model value (δ) for different values of the constant b in the linear confining potential. Here the probability of the $qqqq\bar{q}$ component in both the nucleon and in the $\Delta(1232)$ is taken to be 10%.

b (MeV)	$K(k_\gamma)$	$A_{3/2}$ ($1/\sqrt{GeV}$)	$A_{1/2}$ ($1/\sqrt{GeV}$)	$A_{3/2}/A_{1/2}$	δ
300	0.26	-0.151	-0.086	1.76	1.08
350	0.18	-0.156	-0.088	1.78	1.16
400	0.11	-0.162	-0.090	1.80	1.23
450	0.03	-0.167	-0.092	1.82	1.31
500	-0.04	-0.173	-0.094	1.84	1.39
550	-0.12	-0.178	-0.096	1.86	1.48

confinement, which is consistent with the wave function model. This is obtained by the substitution [9]:

$$cr - b \rightarrow \frac{1}{2}Cr^2 - B\,. \tag{24}$$

Here B is a constant that shifts the interaction potential to negative values at short range. The oscillator constant C is given as [11]

$$C = \frac{m\omega_5^2}{5}\,. \tag{25}$$

With $m = 340$ MeV and $\omega_5 = 245$ MeV this gives for C the value 105 MeV/fm^2.

The helicity amplitude for confinement triggered annihilation $\Delta^+ \rightarrow p\gamma$ in this oscillator confinement model may be obtained directly from the expression for linear confinement above (23) by the substitution [9]

$$cK(k_\gamma) \;\rightarrow\; \frac{\sqrt{6}}{6}\frac{C}{\omega}L(k_\gamma)\,. \tag{26}$$

The function $L(q)$ is defined as the integral

$$L(q) = \sqrt{\pi}\omega_5^5 \int_0^\infty d\xi\, \xi^4 \,\frac{j_1(\beta q\xi)}{\beta q\xi}\, e^{-\alpha^2\xi^2}\,(\frac{3}{4} + \omega^2(\xi^2 - \frac{3B}{2C}))\,. \tag{27}$$

For $\Delta^+(1232) \rightarrow p\gamma$, k_γ=259 MeV and $L(k_\gamma)$=2.0 in the case where $B = 0$.

The confinement triggered annihilation transitions also in this case counteracts the contribution from the direct annihilation transition, unless B takes a very large value. The

TABLE II: Calculated helicity amplitudes, the ratio between the helicity amplitudes and the enhancement of calculated electromagnetic decay width from the qqq quark model value (δ) for different values of the constant B in the harmonic confining potential. Here the probability of the $qqqq\bar{q}$ component in both the nucleon and in the $\Delta(1232)$ is taken to be 10%.

B (MeV)	$L(k_\gamma)$	$A_{3/2}$ $(1/\sqrt{GeV})$	$A_{1/2}$ $(1/\sqrt{GeV})$	$A_{3/2}/A_{1/2}$	δ
50	1.41	-0.157	-0.088	1.78	1.17
100	0.82	-0.162	-0.090	1.80	1.24
150	0.23	-0.168	-0.092	1.83	1.32
200	-0.36	-0.173	-0.094	1.85	1.40
250	-0.95	-0.179	-0.096	1.87	1.49
300	-1.54	-0.184	-0.098	1.89	1.57

contribution from the confinement triggered annihilation to the helicity amplitudes of the $\Delta^+ \to p\gamma$ decay vanishes if B takes the value $B \approx 170$ MeV. For values of B in the range $100 - 200$ MeV, the confinement triggered annihilation transitions are insignificant (Table II).

IV. DISCUSSION

The results obtained above with an extension of the qqq quark model to include $10 - 20\%$ admixtures of the $qqqq\bar{q}$ configurations, that are expected to have the lowest energy, reveal that direct annihilation transitions of the form $qqqq\bar{q} \to qqq\gamma$ significantly reduce the difference between the calculated and the empirical values of the helicity amplitudes for $\Delta \to N\gamma$ decay. As in addition the increase of the calculated $A_{\frac{3}{2}}$ helicity amplitude is larger than that of the $A_{\frac{1}{2}}$ amplitude, the calculated ratio $A_{\frac{3}{2}}/A_{\frac{1}{2}}$ may be brought into agreement with the empirical values by introduction of such $qqqq\bar{q}$ admixtures into the quark model wave functions for the nucleon and the $\Delta(1232)$ resonance.

These results are consistent with the conclusion based on studies of the coupled channel $N - \Delta - \pi$ system, that the effect of the "pion cloud" around the baryons is significant, and responsible for $\sim 30\%$ of the $N - \Delta$ transition magnetic moment.

The change of the calculated ratio of the helicity amplitudes $A_{\frac{3}{2}}/A_{\frac{1}{2}}$ from the quark model value $\sqrt{3}$ to 1.88 is consistent with the magnitude expected on the basis of the large N_C limit of QCD [17]:

$$A_{\frac{3}{2}}/A_{\frac{1}{2}} = \sqrt{3} + O(1/N_c^2)\,. \tag{28}$$

Equivalently, the ratio E_2/M_1 of the multipole amplitudes is predicted to be order $1/N_c^2$.

Introduction of $10-20\%$ admixtures of $qqqq\bar{q}$ components in the wave functions of the nucleon and the $\Delta(1232)$ resonance was however not found to be sufficient to completely remove the underprediction of the empirical values of the helicity amplitudes $A_{\frac{1}{2}}$ and $A_{\frac{3}{2}}$ in the quark model. Additional annihilation mechanisms that are triggered by the interaction between the quarks may be required for this purpose. As an example of such a mechanism the annihilation mechanism that is triggered by the confining interaction was considered here. In the case of linear confinement it was however found that this mechanism only leads to an additional enhancement if the linear interaction potential is large an negative at short range, which may not be phenomenologically realistic.

Acknowledgments

Research supported in part by the Academy of Finland grant number 54038

---

[1] B. Juliá-Díaz, D. O. Riska and F. Coester, Phys. Rev. **C70** (2004) 045205

[2] T. Melde et al., hep-ph/0406023

[3] T. Sato and T.-S. H. Lee, Phys. Rev. **C63** (2001) 055201

[4] T. Sato, D. Uno and T.-S. H. Lee, Phys. Rev. **C67** (2003) 065201

[5] R. S. Towell et al., Phys. Rev. **D64** (2002) 052002

[6] A. Baldit et al., Phys. Lett. **B332** (1994) 244

[7] P. Amaudruz et al., Phys. Rev. Lett. **66** (1994) 2712

[8] K. Ackerstaff et al., Phys. Rev. Lett. **81** (1998) 5519

[9] Q. B. Li and D. O. Riska, nucl-th/0507008

[10] S. Eidelman et al., Particle Data group, Phys. Lett. **542**(2004) 1

[11] C. Helminen and D. O. Riska, Nucl. Phys. **A699**(2002) 624

[12] J.-Q. Chen, Group Representation Theory for Physicists, World Scientific, Singapore (1989)

[13] K. O. Henriksson, T. A. Lähde, C. J. Nyfält and D. O. Riska, Nucl. Phys. **A686** (2001) 355

[14] T. A. Lähde, C. J. Nyfält and D. O. Riska, Nucl. Phys. **A645** (1999) 587

[15] K. O. Henriksson, T. A. Lähde, C. J. Nyfält and D. O. Riska, Nucl. Phys. **A674** (2000) 141

[16] M. Genovese, J. -M. Richard, Fl.Stancu and S. Pepin, Phys. Lett. **B425** (1998) 171

[17] E. Jenkins, X. D. Ji and A..V. Manohar, Phys.Rev.Lett. **89** (2002) 242001